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Kinetic Analysis of Electron Collection to Biased Objects in Magnetized Plasmas

Jin-Hee KIM
System LSI Business, Samsung Electronics, KieHeung 449-711

Kyu-Sun CHUNG*
Department of Nuclear Engineering, Hanyang University, Seoul 133-791

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A one-dimensional collisionless kinetic model is developed for electron collection to a positively biased object in magnetized plasmas. The cross-field flow into the collection tube is modelled by accounting consistently for particle exchange between the collection flux tube and the outer plasma. Numerical solutions of the self-consistent plasma/sheath equations are obtained for a stationary plasma. The spatial dependence of the electron distribution function, its moments (density, particle flux, and temperature), and potential at sheath are obtained.

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I. INTRODUCTION

The space shuttle Orbiter in polar earth orbit sees an environment which differs dramatically from that in equatorial orbit. Near the arctic circle, the Orbiter may experience intense fluxes of energetic (5-10 keV) electrons propagating down along magnetic field lines. The precipitating electrons collide with molecules in the atmosphere to generate the aurora borealis. When they impinge upon the space shuttle they may cause potential differences as large as a few thousand volts [1]. The developed potentials increase rapidly with increasing vehicle dimensions and are extremely sensitive to the ratio of precipitating electron currents to discharging currents, such as those associated with ion collection by the ram effect and secondary electron emission [2].

One of the primary difficulties associated with high voltages on satellites in low earth orbit (LEO) is the power drain due to leakage currents flowing between exposed surfaces through the ambient plasma. The ground potential of an isolated object in space adjusts to collect no net plasma current. However, electrons collected by positive surfaces and positive ions collected by negative surfaces form current loops through the plasma waste solar array power. In order to predict parasitic losses due to a surrounding plasma, a simplified nonlinear screening model for electric potentials and plasma structure surrounding an object with exposed high voltage surfaces was proposed, and simulations were done in a positive bias (electron-collecting) mode [3]. Similar results would be obtained for negative (ion-collecting) objects with currents reduced by the square root of the mass ratio. Since the shuttle is so large that magnetic field effects, although very weak (about 0.5 gauss), can be important, the theory of charged particle collection in magnetized plasmas should be directly applicable. However, some of the general features of our present approach are significant even for the situation in which the magnetic field can be ignored. Thus, our theory may be applicable in part to many problems of the plasma 'wake' [4–6]. The upstream 'ram' [1] tends to be less affected by the self-consistent electric fields and hence is a less difficult problem, from the plasma viewpoint.

The collection of electrons in a strong magnetic field has been a long-standing problem. The problem is to explain why the ratio of the electron saturation current to the ion current is so much smaller in a high magnetic field than in no magnetic field. The problem still arises in fusion devices and in lower earth orbit space vehicles. It is not really clear how to reliably interpret the current-voltage characteristic with regard to electron collection. Bohm's basic idea is that the electrons suffer friction with the ions when one biases the probe to collect electrons [7]. To pull in the electrons, an electric field arises along the magnetic field lines in the flux tube contacting the probing object, and the ions are repelled and are in a static equilibrium. Thus, the ions satisfy a Boltzmann relation, which means there is a potential hill just in front of the probe where $n_e = n_i = n_1$ and $n_1 < n_o$, and $n_1 \ll n_o$ if electron-ion (e-i) friction is very

*E-mail: kschung@hanyang.ac.kr

strong, where no is the unperturbed density. Thus, the electron collection, $n_1 v_e/4$, is reduced compared to the value of $n_o v_e/4$ without a magnetic field. Sanmartin did a high-powered kinetic analysis of the problem assuming only classical cross-field transport and claimed isothermal ions [8]. Cohen followed this with a simpler model and drew figures showing the potential hill [9]. Stangeby developed a fluid model based upon Bohm's idea and analyzed the electron current from negligible levels upto the electron saturation current while Bohm only analyzed the electron saturation current [10]. However, Guenther questioned this entire picture by stating that previous models by Bohm and Stangeby lacked a physical foundation since they neglected e-i collisions and only considered electron-neutral (e-n) collisions, and presented an entirely new model [11]. Although he misunderstood their models, since they included e-i collisions in their models, his contention about the potential hill seems to be correct; *i.e.*, if one considers only e-i collisions, then there is no need for a potential hill. In order to resolve the above argument, one needs to do a kinetic analysis of the electron collection with and without the potential hill assumption and to check the self-consistency.

Our theory is addressed primarily to the case where the magnetic field is strong enough that the electron gyroradius is substantially smaller than the probing object. In this case, electron collection across the field is diffusive even if the parallel flow is dominated by inertial effects. As a result, the quasi-neutral perturbation region (flux tube), where the acceleration of the electrons occurs into the sheath, becomes highly elongated along the field until the cross-field diffusion is able to balance the parallel collection flow. Since the perpendicular momentum is unimportant in this process, it appears attractive to attempt to simplify the problem by treating the flux tube as effectively one dimensional. One can then seek solutions satisfying the Poisson equation and the Boltzmann equation in the parallel direction, treating the perpendicular diffusion equation as a source term in the parallel equations. It is the purpose of this work to analyze the problem by using a one-dimensional collisionless kinetic analysis with isothermal ions and accounting correctly for the diffusive nature of the electron source.

II. MODEL AND ANALYSIS

The one-dimensional equation governing electrons in the flux tube can be described by the Boltzmann equations as

$$\frac{df_e}{dt} = W(z,v)[f_{e^\infty} - f_e(z,v)] \ , \tag{1}$$

where $f_e(z,v)$ is the one-dimensional electron distribution function, z is the position along the magnetic field, v is the parallel velocity of the electrons, and $W(z,v)$ is the frequency of the electron exchange between the collection tube and the outside, which is given by $W(z,v) = D_\perp(z)F(v)$. The energy equation is

$$E = \frac{1}{2}mv^2 - eV \ , \tag{2}$$

where E is the constant total energy, and V is the voltage applied to the probing object. If ions are assumed to be isothermal for a strongly positively bias object described by the Boltzmann relation, then the Poisson equation becomes

$$\frac{d^2V}{dz^2} = 4\pi e n_0 \Big[\int dv f_e(z,v) - \frac{q}{en_0} \exp\Big[\frac{ZeV}{T_i}\Big]\Big] \ , \tag{3}$$

where the boundary conditions are assumed to be i) $f_e(z\to 0, v\geq 0) = 0$, ii) $f_e(z\to\infty, v) = f_\infty(v)$, and iii) $f_e(z, v\to\pm\infty) = 0$. Then, the above equations can be nondimensionalized as :

$$\frac{dg}{du} = \frac{F(u)}{Zd\eta/d\zeta}[g_\infty(u) - g(\zeta,u)] \ , \tag{4}$$

$$\epsilon = \frac{u^2}{2Z} - \tau\eta \ , \tag{5}$$

$$\tau\alpha^2\frac{d^2\eta}{d\zeta^2} = \infty g(\zeta,u)du - Ze^{-\eta} \tag{6}$$

by using the following transformations:

$$\zeta \equiv \frac{z}{L_e} \equiv \frac{2D_\perp}{a^2 v_e}z, v_e^2 \equiv \frac{2T_e}{m_e}, u \equiv \frac{v}{v_e}, Z \equiv \frac{q}{e},$$
$$\tau \equiv \frac{T_i}{ZT_e}, \epsilon \equiv \frac{E}{ZT_e}, \eta \equiv \frac{ZeV}{T_i},$$
$$\alpha \equiv \frac{\lambda_e}{L_e}, g(\zeta,u) \equiv \frac{v_e}{n_0}f(\zeta,u),$$

where λ_e is the electron Debye length, and the normalized boundary conditions are I) $g(\zeta\to 0, u >= 0) = 0$, ii) $g(\zeta\to\infty, u) = g_\infty(u)$, and iii) $g(\zeta, u\to\pm\infty) = 0$. Here, the normalized Maxwellian distribution is given by $g_{infty}(u) = \sqrt{\tau/\pi}\exp[-\tau u^2]$.

Since the ratio (α) of the Debye length to ion collection length is typically $\ll$1, very fine resolution is needed near the sheath region. To provide this without increasing the number of mesh points excessively, we choose a nonuniform meshing along the ζ-direction. If $\zeta = s^\delta$ ($\delta > 1$), Eqs. (4) and (6) become

$$\frac{dg}{du} = \frac{\delta s^{\delta-1}F(u)}{Zd\eta/ds}[g_\infty(u) - g(s,u)] \ , \tag{7}$$

$$\tau\frac{\alpha^2}{\delta^2}\frac{d^2\eta}{ds^2} = \int g(s,u)du - Ze^{-\eta} \ . \tag{8}$$

Then, we use a mesh that is uniform in s. Since $F(u)$ comes from the frequency function $W(\equiv 2D/a^2)$, which depends upon the cross-field diffusivity, and u (or v) is parallel to the flux tube, one can consider $F(u)$ to be independent of u; *i.e.*, we can put $F(u)$ as $F(u) = 1$.

To solve the above equations, we guess an initial potential variation. Along each energy orbit, the velocity sets are obtained as $u_{i,j} = \sqrt{2Z(\epsilon_j + \tau\eta_i)}$, where i is the position index and j is the energy index. We obtain the ion distribution function along the orbits by solving the kinetic equation with a semi-implicit method; *i.e.*,

$$g_{i+1,j} = \frac{[K_i h_{i,j} g_{\infty i,j} + [1-(1-\nu)K_i h_{i,j}]g_{i,j}]}{[1+\nu K_i h_{i,j}]} , \tag{9}$$

where $K_i = \delta s_i^{\delta-1}/(d\eta/ds)_i$, $h_{i,j} = u_{i+1,j} - u_{i,j}$, and ν is a mixing parameter. The boundary conditions are $g_{1,j} = 0$, $g_{N_p,j} = g_{\infty i,j}$. where N_p is the maximum number of meshes in position. Because of the shape of the orbit and the fixed position grid, the velocity spacing between adjacent points on an orbit is large, near $u = 0$. In order to minimize the numerical error which otherwise arises in the orbit integration, we introduce an additional point on the orbit at $u = 0$. The value of potential there is appropriately interpolated between the adjacent points on the position mesh. This greatly improves the accuracy of the distribution function on the negative velocity side. Since Eq. (8) is an elliptic equation, one can solve it by using the successive over-relaxation method [12] for nonuniform meshing in ζ (but uniform meshing s). The scheme for the potential is

$$\eta_i^{\mu+1} = \frac{[-\omega_i A_i \eta_{i-1}^{\mu+1} + (1-\omega_i)B_i\eta_i^{\mu} - \omega_i C_i \eta_{i+1}^{\mu} + \omega_i\sigma_i)}{B_i} , \tag{10}$$

where $B_i = -2(\alpha/\delta\triangle s)^2$ and $A_i = C_i = -0.5B_i[1 + (\delta-1)s_i\triangle s/2]$. Here, σ_i is charge density at position i, replacing the left-hand side of Eq. (8), ω_i is the generalized relaxation parameter, $\triangle s = L/N_p$, where L is the total length of the flux tube, and μ is an iteration index. For the quasineutral case (α=0), Eq. (8) can be used to obtain the potential directly as

$$\eta_i^{\mu+1} = -Zln[n_i(\eta_i^{\mu})] . \tag{11}$$

However, direct iteration schemes of this type are usually unstable [13]. Instead, a relaxation scheme has been used:

$$\eta_i^{\mu+1} = \eta_i^{\mu} + \beta ln(n_e/n_i) , \tag{12}$$

where β is a relaxation parameter ($0 < \beta < 1$). We obtain self-consistent solutions for the potential and the ion distribution function by iterating these procedures until they reach a convergence criterion, *i.e.*, $\max|\eta_i^{\mu+1} - \eta_i^{\mu}| < \epsilon$. After the self-consistent ion distribution and potential variation are gotten, the moments of the ion distribution (density, flux, velocity, temperature) are obtained by using Simpson's rule to integrate of the ion distribution over the nonuniform velocity space at each position.

III. RESULTS

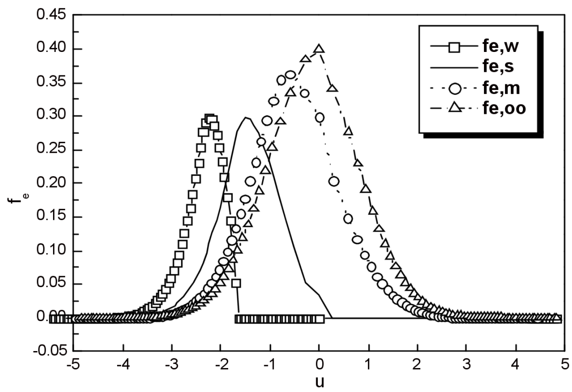

Fig. 1. Electron distributions at different potentials and, hence, positions along the flux tube for η_w=3. : w=wall position, s=sheath position, m=middle position $x = x(\eta = \eta_w/2)$, ∞=end of the electron flux tube (η=0).

For the following results, we use singly charged ions, $Z = 1$, $\tau = 1(T_e\ infty = T_i)$, nonuniformity of distance, $\delta = 2.5$, a convergence criterion $\epsilon{\sim}10^{-4}$, a position mesh N_p=51, and an energy mesh N_e=100. Figure 1 shows the ion distributions at different potentials and, hence, positions along the flux tube. We find that the potential perturbation is noticeably larger; consequently, the density falls off toward the probe rapidly. In order to accelerate the electrons to hit the biasing surface faster than the thermal speed, a greater potential drop is required than if the momentum loss is ignored. Conventionally, the 'sheath edge' is defined at the position where the ion fluid velocity approaches the ion sound velocity ($v_s \equiv \sqrt{(T_e+T_i)/m_i}$), when a floating surface is negatively charged due to the fast isothermal motion of electrons. However, it is not so clear whether one can adopt the same philosophy for the case of electron acceleration by a positively biased surface. Since the electron debye length is so small compared with the characteristic length of the electron, *i.e.*, $\alpha(=\lambda_e/L_e) \approx 0$, one can consider the electron sheath to be at a position next to the wall, *i.e.*, $x_s = x(i$=2).

The distribution functions at x_s, with different potentials applied to the wall, are shown in Fig. 2. As the wall potential gets larger, the electron distribution at the position (f_{es}) becomes narrower and smaller. Also the electron fluid velocity becomes larger with the applied wall potential. The different total heights show the density reduction when the wall potential gets larger; *i.e.*, the wall (sink) draws the electrons more strongly. For the purposes of interpreting the interaction of the plasma with material objects, it is the values of the parameters at the wall (material surface) that matter most. However, some of these values depend on the wall potential, so it is difficult to give compact general results for the wall value. Adopting a wall potential that gives zero total electric current (the floating potential) is a special value that is often adopted, but is by no means generally appropriate. Our approach is to take advantage of the

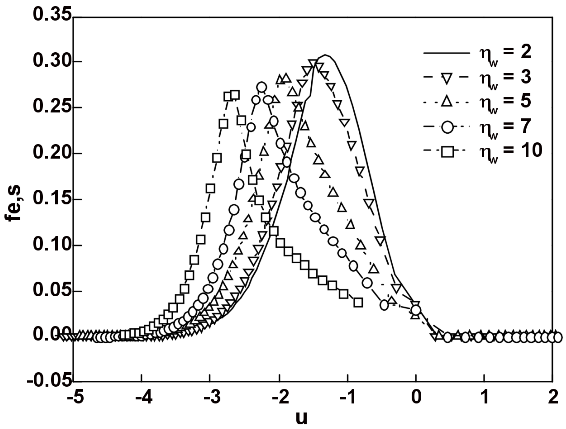

Fig. 2. Distribution functions at the 'electron sheath edge,' *i.e.*, $x = x(i{=}2)$ with different potentials applied to the wall.

fact that for $\alpha \ll 1$, the sources within the sheath are negligible. Therefore, the values of the electron parameters at the wall are related to those at the sheath edge via a trivial transformation: an energy- and flux-conserving fall through a potential drop equal to the difference between wall and the sheath potentials. This means, of course, that the wall electron flux is the same as the flux at the sheath edge. Other parameters for any wall potential (more negative than the sheath potential) can be calculated from the sheath values that we give.

Figure 3 gives the moments of the electron distribution (flux, temperature) and the potential at the sheath edge. The sheath potential is important in defining the sheath potential drop for a given wall potential (relative to the plasma). The electron temperature at the sheath has larger values than outside the flux tube. This is an interesting result because most previous fluid treatments assume that there is no temperature gradient along the flux tube. Our results show that this is a bad approximation. The electron flux seems to be constant for various wall potentials while sheath potential tends to increase slowly with the wall potential.

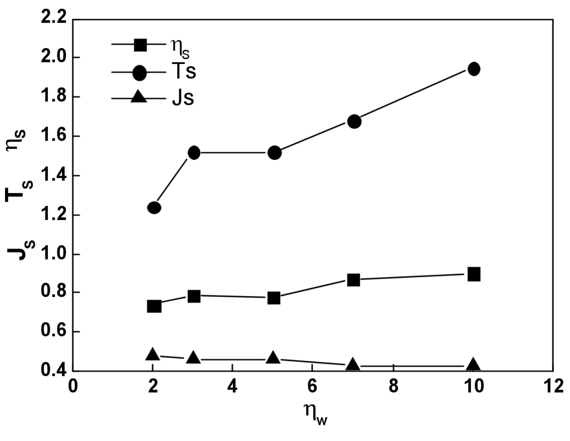

Fig. 3. Moments of the electron distribution (flux, temperature) and potential at the sheath edge.

IV. CONCLUSION

We have developed a one-dimensional kinetic model for the flow of electrons to a positively biased object in a magnetic field. We use an electron source term along the perturbation region that treats consistently the diffusive cross-field fluxes. The electron parameters, such as distribution function, particle flux, temperature, and potential have been obtained. The potential and the flux seem to be constant, regardless of the wall potential. The electron temperature variation with the wall potential is not constant. However, it must be recalled that we are treating a very simplified one-dimensional model. Multi-dimensional effects in the wake may reveal much more complex physical structures.

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