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Terrestrial Application of Gasdynamic Mirror Fusion

Myoung-Jae Lee*
Department of Physics, Hanyang University, Seoul 133-791

Kyu-Sun Chung
Department of Nuclear Engineering, Hanyang University, Seoul 133-791

Terry Kammash
Department of Nuclear Engineering and Radiological Sciences, University of Michigan, Ann Arbor, MI 48109, U.S.A.

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Fusion gasdynamic mirror (GDM) concepts have been previously proposed and explored for space propulsion systems. The GDM is a device with a very large aspect ratio which confines a hot plasma long enough to generate fusion energy. By noting that the confinement of a GDM is governed by gas dynamic laws and using an appropriate set of particle and energy balance equations for the D-T fuels and reaction products, the GDM plasma has been shown to be able to obtain a large fusion gain factor Q (ratio of fusion power to injected power) not attainable with traditional mirror fusion reactors. For an example, the GDM has been shown to have a capability of producing a Q value of 30 for a reasonable plasma length of 30 meters with an injection energy of 20 keV and a mirror ratio of 50. In this case, the steady-state ion density and temperature have been found to be 1.6×10^{17} cm^{-3} and 37 keV, respectively. The merit of a high-Q value certainly provides the possibility of replacing conventional mirror fusion reactors for electric power production.

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I. INTRODUCTION

Traditional fusion research efforts [1–3] have discarded magnetic mirrors as a practicable option due to the plasma losses from the open configuration. Fortunately, this loss phenomenon has been adopted as a viable space propulsion driver and has been studied extensively in recent years [3–8]. Especially, taking advantage of this, Kammash *et al.* have proposed the concept of a gasdynamic mirror (GDM) fusion propulsion system [9–13]. The GDM is a mirror device with very large aspect ratio ($L/r_p \gg 1$, where L is the plasma length and r_p the plasma radius) which confines a hot plasma long enough to generate fusion energy while allowing a certain fraction of its charged particles to escape from one end to produce thrust [9]. In a GDM, the plasma will have a density and temperature that make the ion-ion collision mean free path (λ) much shorter than the plasma length (L) of the mirror. Under these conditions, the plasma behaves like a continuous medium - a fluid, and its confinement is governed by gasdynamic laws. This gasdynamic property of a mirror plasma enables us to obtain a large enough Q-value (ratio of fusion power to injected power) which is not attainable with traditional mirror fusion reactors. In this paper, it will be shown that the GDM concept as a terrestrial fusion reactor application for energy production, as well as its application to a plausible space propulsion system, appears to be promising.

In contrast with the classical terrestrial mirror device where $\lambda \gg L$, the confinement time appears as a linear function of R rather than a logarithmic function because the plasma flow in a GDM is governed by the gasdynamic equations. Therefore, the particle lifetime in a GDM is expressed as

$$\tau = \frac{RL}{v_{th}}, \tag{1}$$

where R is the mirror ratio seen by the plasma and and v_{th} is the thermal velocity defined as the square root of the ratio of temperature to mass. Consequently, in a gasdynamic confinement system, increasing the mirror ratio to the highest value permitted by technology provides a much greater effect than in a classical mirror. For reasons that will become clear shortly, we consider in this analysis systems for which $R \gg 1$. It might be added at this point that when $R \gg 1$, the condition for the validity of Eq. (1) is $\lambda/R \ll L$, which is much less

*E-mail: mjlee@hanyang.ac.kr

stringent than the condition $\lambda \ll L$ [14]. In other words, the quantity to be compared with the length of the device, L, is not the mean free path, λ, but an effective mean free path λ/R against scattering through an angle of the order of the loss cone angle $\theta = \sin^{-1}(1/\sqrt{R})$. This angle represents a region in velocity space which if a particle falls in it as result of a Coulomb collision with another particle, it will definitely escape through the end. Under these conditions, any loss cone instability arising from depletion of the velocity distribution function will not have an important effect on the longitudinal confinement time [15]. Since this instability is associated with electrostatic waves that propagate nearly perpendicular to the magnetic field, any reduction in this activity will also result in a reduced particle transport across the magnetic field, *i.e.*, improved radial confinement. Moreover, no reduction in the scattering length can cause a significant change in the rate at which the plasma is lost through the mirrors; thus, it cannot lower the lifetime below the value given in Eq. (1). Another important factor of this system is the high plasma density in the region just beyond the mirror where it is comparable to that at the center of the system. This will allow magnetohydrodynamic stability and make it possible to suppress the flute instability, which is known to plague classical axisymmetric mirrors [14,16,17].

We shall focus on a deuterium-tritium (D-T) fuel cycle at a 50 %-50 % mixture where the reaction products are neutrons and alphas particles. Since the neutrons escape the plasma while the alpha particles remain to deposit their energy in it, the dynamic equations we employ represent the temporal change of the density and the energy of all the charged particle species that make up the plasma; namely, the fuel ions, the electrons and the alpha particles. For simplicity, we will use one ion species with an average mass of 2.5 atomic mass units and further assume that charge neutrality prevails throughout the system at all times. We shall also assume a high-β (ratio of plasma pressure to magnetic field pressure) operation of the system arising from a large degree of gross plasma stability. As noted earlier, this stability comes about as a result of good magnetic field curvature over most of the region occupied by the plasma, including that which is just outside the mirror. Moreover, a high-β operation allows us to assess the impact of reduced synchrotron radiation losses, especially in cases where high-reflectivity (R_e) walls are assumed.

II. PLASMA DYNAMICS

A point-reactor model is employed, where we assume that the plasma density and energy are nearly uniform throughout the system. The reaction of deuterium and tritium, *i.e.*,

$$D + T \rightarrow He^4(3.52\ \text{MeV}) + n(14.07\ \text{MeV}),$$

is considered since it is one of the most important fusion fuel cycles for space and terrestrial applications. If we denote the fuel ion (50 %-50 % D-T) density and energy by n_i and E_i, respectively, then the mass and energy conservation equations for this species can be written as

$$\frac{dn_i}{dt} = S_i - \frac{n_i}{\tau_i} - \frac{1}{2} n_i^2 \langle \sigma v \rangle_{DT} \tag{2}$$

and

$$\frac{d(n_i E_i)}{dt} = S_i E_{in} - \frac{n_i}{\tau_i} E_{Li} - \frac{1}{2} n_i^2 \langle \sigma v \rangle_{DT} E_i + W_{f\alpha i} + W_{ei} + W_{\alpha i}. \tag{3}$$

In these equations, S_i is the injection rate of fuel ions per unit volume, and E_{in} is the energy with which they are injected into the plasma. τ_i is the lifetime or confinement time of the ions, and E_{Li}, given by $2T_i$ where T_i is the ion temperature, represents the energy with which they escape confinement through the mirror. The quantity $\langle \sigma v \rangle_{DT}$ is the velocity average of the fusion cross-section, and the last term in Eq. (2) represents the fusion reaction rate where two fuel ions are removed per reaction. The corresponding term in Eq. (3) denotes the energy loss due to the ions that undergo a fusion reaction since each carries an energy E_i. The last three terms in Eq. (3) denote, respectively, the rate of energy transfer to the fuel ions from the fast alpha particles ($W_{f\alpha i}$) generated by the D-T fusion reactions, from the electrons (W_{ei}), and from the thermal alpha particles ($W_{\alpha i}$). We distinguish between the fast alphas or the high-energy group, and those which have reached thermalization, and as a result, are characterized by an appropriate temperature. In a similar manner, we write the conservation equations for the electrons in the plasma; they are

$$\frac{dn_e}{dt} = S_i - \frac{n_e}{\tau_e} \tag{4}$$

and

$$\frac{d(n_e E_e)}{dt} = S_i E_{in}^e - \frac{n_e}{\tau_e} E_{Le} + W_{f\alpha e} + W_{\alpha e} - W_{ei} - P_b - P_s, \tag{5}$$

where we have let the injection rate S_e be the same as that for the ions, *i.e.*, S_i. The terms in these equations are defined exactly as those in Eqs. (2) and (3) except for P_b and P_s which represent the bremsstrahlung and synchrotron radiation loss terms, respectively. The first one, *i.e.*, P_b can be written as

$$P_b = p_0 n_e (n_i + 4n_\alpha + 4n_{f\alpha}) T_e^{1/2}\ [\text{keV}\cdot\text{cm}^{-3}\cdot\text{sec}^{-1}], \tag{6}$$

where $p_0 = 3.34 \times 10^{-15}$ keV$^{1/2}\cdot$cm$^3\cdot$sec^{-1} is a constant [18], while the second term can be expressed in a relativistically correct form as [19]

$$P_s = \frac{3.12 \times 10^{-16}}{\beta} n_e^2 T_e^2 \left(1 + \frac{T_e}{204}\right) \times (1 - R_e)\ [\text{keV} \cdot \text{cm}^{-3} \cdot \text{sec}^{-1}]. \tag{7}$$

In Eqs. (6) and (7), the temperatures and densities are measured in keV and cm^{-3}, respectively. Equation (7) reveals vividly the reduction in the radiation power as $\beta \rightarrow 1$ because, in this case, the plasma effectively shields the electrons from the magnetic field. Moreover, we observe that a perfectly reflecting wall will completely prevent the escape of this radiation from the plasma and allow it to be absorbed and, in turn, cause an increase in the plasma temperature.

The remaining conservation equations in our model belong to the thermal alpha particles; they are

$$\frac{dn_\alpha}{dt} = \frac{1}{4}n_i^2\langle\sigma v\rangle_{DT} - \frac{n_\alpha}{\tau_\alpha} \tag{8}$$

and

$$\frac{d(n_\alpha E_\alpha)}{dt} = \frac{1}{4}n_i^2\langle\sigma v\rangle_{DT}E_\alpha - \frac{n_\alpha}{\tau_\alpha}E_{L\alpha} + W_{f\alpha\alpha} - W_{\alpha i} - W_{\alpha e}, \tag{9}$$

where τ_α and $E_{L\alpha}$ are defined similarly with those given in Eqs. (2) and (3) but are modified to apply to the alpha particle. In order to compute the energy transfer rate between the various species, *i.e.*, the W terms that appear in the above equations, we begin by writing the following generic equation that represents the rate of energy loss by a particle of mass m_1 and charge e_1 as it interacts with a target particle of mass m_2 and charge e_2 through a Coulomb interaction. The energy transfer from fast alphas to electrons was previously obtained in the form [20]

$$W_{f\alpha e} = \frac{64\sqrt{\pi}n_e e^4 m_e^{1/2} n_{f\alpha}E_{f\alpha}}{3\sqrt{2}m_\alpha T_e^{3/2}}\ln\Lambda_{f\alpha e}. \tag{10}$$

We obtain the portion deposited in the ions and thermal alphas together from the fast alphas, *i.e.*,

$$W_{f\alpha(i+\alpha)} = \frac{n_{f\alpha}}{\tau_{f\alpha}}(E_0 - E_{th}) - W_{f\alpha e}, \tag{11}$$

which upon apportionment yields

$$W_{f\alpha\alpha} = \frac{4n_\alpha \ln\Lambda_{f\alpha\alpha}}{\frac{n_i m_\alpha}{m_i}\ln\Lambda_{f\alpha i} + 4n_\alpha\ln\Lambda_{f\alpha\alpha}}\left[\frac{n_{f\alpha}}{\tau_{f\alpha}}(E_0 - E_{th}) - W_{f\alpha e}\right], \tag{12}$$

$$W_{f\alpha i} = \frac{\frac{n_i}{m_i}\ln\Lambda_{f\alpha i}}{\frac{n_i}{m_i}\ln\Lambda_{f\alpha i} + \frac{4n_\alpha}{m_\alpha}\ln\Lambda_{f\alpha\alpha}}\left[\frac{n_{f\alpha}}{\tau_{f\alpha}}(E_0 - E_{th}) - W_{f\alpha e}\right]. \tag{13}$$

As for the rate of energy exchange between the thermal species of kind j and k, the standard formula can be readily applied, *i.e.*, [18]

$$W_{jk} = \frac{n_j n_k}{(n\tau)_{jk}}(E_j - E_k) \tag{14}$$

with

$$(n\tau)_{jk} = \frac{\sqrt{2}m_j m_k}{8\sqrt{\pi}e^4}\left(\frac{T_j}{m_j} + \frac{T_k}{m_k}\right)^{3/2}\frac{1}{\ln\Lambda_{jk}}. \tag{15}$$

At this moment, it is desirable to refine the confinement times for the electrons and ions without which the conservation equations presented above cannot be solved. We have seen earlier that Eq. (1) would be perfectly applicable to the ions if there were no electrons in the plasma, but because of the presence of this specie an electrostatic potential is created that would make the loss rate for both species the same. This ambipolar loss has been treated many times for the case of the conventional low-density mirror. Here, we incorporated it into the confinement time of a high-density plasma. If we put the result of Eq. (1) into a set of appropriate diffusion equations, we can obtain the ambipolar diffusion coefficient, which in turn allows us to compute the following confinement time for both species:

$$\tau = \frac{RL}{v_{thi}}\left(1 + \frac{4c_0E_e^{5/2}}{3m_i v_{thi} n_e L\ln\Lambda}\right)^{-1}. \tag{16}$$

Here, m_i refers to the ion mass, c_0 is a constant, and $\ln\Lambda$ is the familiar Coulomb logarithm.

III. TERRESTRIAL APPLICATION

When the confinement time given by Eq. (16) is substituted into the standard definition of Q, the ratio of fusion power to injected power, it can readily be shown that

$$Q \approx \frac{nRL\langle\sigma v\rangle}{E_i^{3/2}}. \tag{17}$$

Thus, increasing the mirror ratio R to the highest value permitted by the technology provides a much higher Q in a GDM than in a classical collisionless mirror where the lifetime is a logarithmic function of R. Moreover, as seen in Eq. (17), Q increases with L, providing yet another device parameter that can be used to enhance

Table 1. Reactor parameters of a high-density D-T mirror (E_{in} = 20 keV, β = 0.95).

Parameter	L = 50 m	L = 75 m	L = 100 m	L = 100 m
Gain Factor Q	30	30	30	50
Ion Density n_i (cm^{-3})	1.61×10^{17}	1.07×10^{17}	8.06×10^{16}	1.53×10^{17}
Mirror Ratio R	50	50	50	50
Plasma Radius r_p (cm)	7	7	7	7
Ion Temperature T_i (keV)	36.85	36.82	36.80	54.92
Electron Temperature T_e (keV)	30.11	30.14	30.17	40.02
Plasma Confinement Time τ (sec)	1.19×10^{-3}	1.78×10^{-3}	2.38×10^{-3}	1.95×10^{-3}
Effective Mean Free Path λ/R (m)	3.44	5.17	6.86	8.04
Central Magnetic Field B_{p0} (tesla)	80.21	65.41	56.60	94.08
Fusion Power P_f (MW)	1.10×10^{7}	7.32×10^{6}	5.49×10^{6}	2.22×10^{7}

this quantity. A large aspect ratio mirror ($L/r_p \gg 1$) is particularly desirable not only because of the large Q it can produce, but also because of the MHD stability it provides as a result of the nearly uniform magnetic field and the presence of a high density plasma just beyond the mirror. The conservation equations, Eqs. (2) $\sim$ (5), (8) and (9), including thermal and fast alpha particle effects, are solved numerically to obtain the steady-state reactor parameters. In this analysis, we have assumed a high-density mirror reactor that burns a 50 %-50 % DT mixture. Some plasma parameters for steady-state reactor operation are shown in Table 1. We see that when R and Q are fixed, the density decreases linearly as L increases while both the electron and the ion temperatures remain almost the same, as manifested in Eq. (17). We also observe that the vacuum magnetic field at the center, B_{p0}, varies as the square root of the density. It can be seen that the effective mean free path of ions is much shorter than the length of the GDM system.

In obtaining these results, we have used a β of 0.95 commensurate with a high degree of MHD stability, thus making the vacuum mirror ratio significantly smaller than those given in the table. Though unoptimized, these preliminary results clearly indicate the suitability of this concept for a power reactor, with the observation that the longer the device, the less stringent are the technological requirements, *e.g.*, magnetic field, fueling rate, density, *etc.*, as shown in the first three columns. On the other hand, it is clear from the last two columns that increasing the Q-value for a fixed reactor size, places a more stringent requirement on these parameters.

IV. CONCLUDING REMARKS

We have examined in this paper a magnetic confinement fusion system, which originally appeared to lend itself nicely to a propulsion system that could meet the stringent requirements of space exploration, as a potential terrestrial power plant. One of the major difficulties encountered by the conventional mirror fusion reactor is its inability to produce a large enough Q-value to compensate for the inefficiencies of the various components of the power plant. In such mirrors, a typical fuel ion will traverse the device several times before it undergoes a scattering collision, and the end losses are sufficiently severe to precipitate low Q-values. The gasdynamic mirror, where the mean free path is significantly shorter than the length of the system, has the potential of overcoming these difficulties. A very large aspect ratio device is particularly desirable not only because of better plasma confinement, which allows for more effective energy production, but also because of the hydrodynamic stability it provides as a result of the nearly uniform magnetic field and the presence of a high-density plasma just beyond the mirror. This allows for the utilization of a high plasma pressure (or density) in the system, as reflected by the quantity β, which in turn leads to a reduction of the synchrotron radiation emitted by the hot plasma. In contrast to the traditional collisionless mirror machine, the plasma lifetime in a GDM increases linearly (as opposed to logarithmically) with the magnetic mirror ratio and almost linearly with the length of the device, thereby providing parameters that can be used to enhance the performance of the reactor. Of course, one can argue that injection of higher densities (at desirable temperatures) means larger injection power, so the gain factor may not improve. However, higher densities also mean higher fusion power (due to dependence on the square of the density), and as a result, for certain carefully chosen parameters, the gain factors should increase substantially, thus making the gasdynamic mirror a particularly attractive candidate for a fusion power reactor. This result shows that not only does the advanced space propulsion device appear plausible, but the potential use of this concept as a terrestrial fusion reactor application also appears to be promising. Although the physics modeling of a GDM presented in this paper is very simple, the results are comprehensive and quite indicative of terrestrial applicability.

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REFERENCES

[1] J. L. Bromberg, *Fusion* (MIT Press, Cambridge, 1982).
[2] J.-S. Ko, D.-K. Kim, S. H. Hong and K.-H. Im, J. Korean Phys. Soc. **41**, 212 (2002).
[3] O. Kwon, J. Korean Phys. Soc. **42**, 118 (2003).
[4] R. F. Post and J. F. Santarius, Fusion Tech. **22**, 13 (1992).
[5] J. F. Santarius and B. G. Logan, J. Propulsion and Power **14**, 519 (1998).
[6] B. N. Cassenti, AIAA 2003-4523, *39th AIAA/ ASME/SAE/ASEE Joint Propulsion Conference* (Huntsville, AL, July 20-23, 2003).
[7] F. R. Chang-Diaz, E. Braden, I. Johnson, M. M. Hsu and T. F. Yang, NASA Technical Paper 3359 (1995).
[8] P. G. Mikellides, P. J. Turchi and I. G. Mikellides, J. Propulsion and Power **18**, 146 (2002).
[9] T. Kammash and M.-J. Lee, J. Propulsion and Power **11**, 3 (1995).
[10] T. Kammash, M.-J. Lee and D. I. Poston, J. Propulsion and Power **13**, 421 (1997).
[11] T. Kammash and D. L. Galbraith, J. Propulsion and Power **14**, 24 (1998).
[12] T. Kammash, *50th International Astronautical Congress* (Amsterdam, Netherlands, 1999), IAF-99-S.6.01.
[13] T. Kammash, *A Design Study of a p-B^{11} Gasdynamic Mirror Fusion Propulsion System, Space Technology and Applications International Forum* (Albuquerque, NM, Feb. 2-5, 2003).
[14] V. V. Mirnov and D. D. Ryutov, Pis'ma Zh. Tekh. Fiz. **5**, 678 (1979) [Translation: Soviet Tech. Phys. Lett. **5**, 279 (1979)].
[15] R. F. Post, Nuclear Fusion **27**, 1579 (1987).
[16] V. P. Nagornyj, D. D. Ryutov and G. V. Stupakov, Nuclear Fusion **24**, 1421 (1984).
[17] M. N. Rosenbluth and C. L. Longmire, Ann. Phys. **1**, 120 (1957).
[18] T. Kammash, *Fusion Reactor Physics: Principles and Technology* (Ann Arbor Science Publishers, Ann Arbor, 1975), Chap. 4.
[19] D. P. Kilcrease and R. C. Kirkpatrick, Nuclear Fusion **28**, 1465 (1988).
[20] T. Kammash and D. L. Galbraith, Nuclear Fusion **13**, 133 (1973).