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This paper extends the previous kinetic model [Phys. Rev. A **38**, 4721 (1988)] of probing objects in flowing magnetized plasmas by generalizing cross-field transport and adding ionization to the source in the Boltzmann equation along the presheath. Ion sheath current density and ratio (R) of upstream to downstream current are obtained as a function of normalized plasma drift velocity (M), equivalent viscosity ratio (α), ion temperature ($T_{i\infty}$), and ionization rate (σ_i). The form $R = \exp(M/M_c)$ fits the results very well and the calibration factor (M_c) is obtained. Here M_c decreases as α increases, $T_{i\infty}$ increases, or σ_i increases. Comparisons with fluid and kinetic models are presented.

I. INTRODUCTION

The analysis of the plasma boundary layer is important for a wide range of applications such as electrodes in gas discharges, Langmuir probes, plasma–wall interactions in fusion devices, plasma processing, and interaction of rapidly moving bodies with plasma in space.

For a negatively biased absorbing surface, electrons are approximately governed by the Boltzmann relation. Then the remaining question is the ion behavior. The characteristics of potential and ion distribution along the region of perturbation due to an object, or at its surface, strongly depend upon the effective sources within that region. Types of sources include collisions between ions or ions and electrons,[1,2] charge exchange,[3,4] ionization,[5-7] secondary-electron emission,[8] and cross-field transport.[9-13] The form of the plasma source in velocity space along the presheath (including the sheath) is very important in determining the sheath characteristics,[3,4,6,7] even though its spatial shape and collisionality along the presheath do not affect the sheath values such as sheath potential and current density.[2,5,12,13]

The purpose of this work is to extend our previous kinetic theory[13] of the presheath governed by cross-field transport, by generalizing the source term in the Boltzmann equation. In our previous model, we introduced a source that models diffusive exchange of particles between the presheath and the outer plasma. This source is equivalent to taking the ratio of cross-field viscosity to diffusivity (α)[12] as one. Here we not only vary α from zero to one, but also add an ionization term. The inclusion of particle sources corresponding to ionization may be important in magnetized plasma with highly populated neutral background such as plasma processing and the scrape-off layer (SOL) of the tokamak edge, especially for the hydrogenic plasma in the complex SOL.[14,15] Then the effects of viscosity, ionization, and ion temperature on the plasma drift velocity diagnosis are explored.

In Sec. II, we set up a kinetic model which contains "transport" and "ionization" terms as sources. Section III deals with the analyses used to solve the plasma-sheath and plasma equation. In Sec. IV, we present and discuss our results, which include ion distributions, ion currents, and the ratio of upstream to downstream current along the presheath and at the sheath. Section V summarizes the results and discussions.

II. MODEL

A. Governing equations

We approximate the presheath as one dimensional with the ion distribution governed by the Boltzmann equation:

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} + a_z \frac{\partial}{\partial v_z}\right) f(z,v_z,t) = C_f + S_f, \qquad (1)$$

where $f(z,v_z,t)$ is the one-dimensional ion distribution function, z is position along the magnetic field, v_z is the ion velocity parallel to the B field (hereafter we omit the subscript z), a_z is the acceleration of the ions governed by the Lorentz force, C_f is the collision operator, and S_f is the volume source of ions. Assuming steady state ($\partial/\partial t = 0$), and ignoring Coulomb collisions along the presheath ($C_f = 0$), the Boltzmann equation reduces to

$$\left(v \frac{\partial}{\partial z} - \frac{q}{m} \frac{d\phi}{dz} \frac{\partial}{\partial v}\right) f(z,v) = S_f. \qquad (2)$$

Here m, q, and ϕ are ion mass, ion charge, and electric potential, while S_f will be taken as the ion source due to cross-field transport and ionization.

The energy equation, governing the phase space orbits, is

$$\tfrac{1}{2}mv^2 + q\phi(z) = E, \qquad (3)$$

where E is the constant total energy.

The electrons are assumed to be isothermal, described by the Boltzmann relation:

$$n_e(z) = n_\infty \exp[e\phi(z)/T_e], \qquad (4)$$

where n_∞ and T_e are electron density and temperature, respectively, and subscript infinity refers to values outside the presheath.

The electron and ion densities are related by Poisson's equation, i.e.,

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$$\frac{d^2\phi}{dz^2} = -4\pi\left(q\int f(z,v)dv - en_e(z)\right). \tag{5}$$

B. Source

The source term is composed of cross-field transport (S_t) and ionization (S_i).

The cross-field transport is considered to be governed by a "frequency" $W(z,v)$ which gives the rate at which particles are exchanged between the presheath and the outer plasma, so the source due to perpendicular transport becomes

$$S_t = W(z,v)\{\alpha[f_\infty(v) - f(z,v)] + (1-\alpha)[1 - n(z)/n_\infty]f_\infty\}, \tag{6}$$

where $f(z,v)$ is the ion distribution function along the presheath and $f_\infty(v)$ is ion distribution outside of the presheath, which includes the drift velocity (V_d). The first term provides a diffusive type of source, the second a convective type. The reason we introduce these two types of transport source is that we would like to see the effect of each term separately and compare the results with those from fluid models which adopt these terms separately. The idea here is that there is a certain amount of particle exchange, represented by the first term that we have used before,[13] plus a certain amount of particle inflow, represented by the second term. The inflow is presumably caused by the fact that the density is different inside the collection tube, so it is proportional to the density difference. The distribution of the inflowing particles is that of the external plasma and contains the information on the drifting velocity of the bulk plasma.

The coefficient α is the relative weight of diffusive contribution and it turns out to be equivalent to the ratio of parallel cross-field viscosity to cross-field diffusivity when discussing the fluid equations. These can be obtained by taking moments of the Boltzmann equation (2) with (6).[16] Obviously, $\alpha = 0$ corresponds to pure convection (no viscosity) and $\alpha = 1$ to "pure" diffusion. The present model thus allows a direct comparison of a full one-dimensional kinetic analysis with the more approximate fluid analysis[12] of the presheath.

The rate of particle and momentum exchange between the outside and inside of the presheath is related via

$$W \sim D_\perp / a^2, \tag{7}$$

to $D_\perp$, the anomalous cross-field diffusion coefficient, and to a, the characteristic size of the probing object. This identification is the heart of the one-dimensional approximation of the inherently two-dimensional problem. The exact coefficient of proportionality cannot be specified without a two-dimensional solution. However, its magnitude affects only the spatial extent of the presheath, not the fluxes to the probe.

The ionization can be taken as

$$S_i(z,v) = \langle\sigma v\rangle_{\text{ion}} n_e(z) f_n(z,v), \tag{8}$$

where $\langle\sigma v\rangle_{\text{ion}}$ is ionization rate, n_e is electron density, and f_n is the distribution of neutral particles. Assuming f_n to be Maxwellian, if $n_e(z)\langle\sigma v\rangle_{\text{ion}} \propto |v|$, we recover the same source term as Emmert *et al.*,[6] and if $\langle\sigma v\rangle_{\text{ion}} = \text{const}$, we can recover that of Bissel and Johnson.[7]

Then the total source term may be written

$$S_f = \sigma_t S_t + \sigma_i S_i, \tag{9}$$

where $\sigma_t + \sigma_i = 1$ and these are ratios of contribution to the source due to transport (σ_t) and ionization (σ_i), respectively.

III. ANALYSES

If we assume that $W(z,v)$ is independent of v, the equations can be nondimensionalized by using the following transformations. We define a characteristic length as

$$L_\parallel \equiv V_s / W(z),$$

where $V_s = (T_e/m_i)^{0.5}$ is the ion sound speed (ignoring T_i). This is the characteristic length of the presheath, but in general varies with parallel position if W does. Then the nondimensional forms of the parameters are

$$\begin{aligned} &x \equiv \int \frac{W(z)}{V_s} dz, \quad \lambda(x) \equiv \frac{\lambda_D}{L_\parallel(x)}, \quad Z \equiv \frac{q}{e}, \\ &u \equiv \frac{v}{V_s}, \; U_d \equiv \frac{V_d}{V_s}, \; \eta \equiv -\frac{e\phi}{T_e}, \\ &\gamma(x,u) \equiv \frac{\langle\sigma v\rangle_{\text{ion}} n_e(z)}{W(z)}, \; \epsilon \equiv \frac{E}{ZT_e}, \\ &g(x,u) \equiv \frac{V_s}{n_\infty} f(z,v), \; n(x) \equiv \frac{n_i(z)}{n_\infty}, \end{aligned} \tag{10}$$

where λ_D is the Debye length and V_d is the external drift velocity. In terms of these parameters the orbit and Poisson equations for both cases become

$$\epsilon = (u^2/2) - \eta, \tag{11}$$

$$\lambda^2(x)\frac{d^2\eta}{dx^2} = Z\int g(x,u)du - e^{-\eta(x)}. \tag{12}$$

The kinetic equation is given by

$$\begin{aligned} &\left(u\frac{\partial}{\partial x} + \frac{d\eta}{dx}\frac{\partial}{\partial u}\right)g(x,u) \\ &\quad = \sigma_t\{\alpha[g_\infty(u) - g(x,u)] \\ &\qquad + (1-\alpha)(1-n)g_\infty\} + \sigma_i\gamma(x,u)g_n, \end{aligned} \tag{13}$$

where g_∞ is the ion distribution function outside the presheath and g_n is the distribution of neutral particles, which may not have the same temperature as g_∞.

If the external ion distribution is Maxwellian with temperature $T_{i\infty}$, shifted by a drift velocity V_d, then

$$g_\infty(u) = \sqrt{(ZT_e/2\pi T_{i\infty})}\exp[-ZT_e(u-U_d)^2/2T_{i\infty}]. \tag{14}$$

Similarly, the neutral gas distribution is given by

$$g_n(u) = \sqrt{(ZT_e/2\pi T_n)}\exp(-ZT_e u^2/2T_n). \tag{15}$$

The boundary conditions on the distribution function are

$$g(x=0, u \geqslant 0) = 0, \quad g(x=\infty, u) = g_\infty(u), \tag{16}$$

which means that the probe has a perfectly absorbing surface and the ion distribution has the same form as the external plasma at large parallel distance. The boundary conditions on the potential are

$$\eta(x=0) = n_w, \quad \eta(x=\infty) = 0. \tag{17}$$

Since the governing equations have the same forms as those we used in our previous work,[13] we simply follow the same procedures as previously, which are briefly: a semi-implicit method for the kinetic equation integration along the particle orbit and successive overrelaxation method for the plasma-sheath equation, or a simple relaxation method for the plasma equation, with nonuniform meshing in position and velocity space.

IV. RESULTS AND DISCUSSIONS

In the following we restrict our attention to results for cases where $Z = 1$ (singly charged ions), $\lambda = 0$ (quasineutral case, sheath thickness negligible), and $\eta_w = 3$ (probe bias potential). The presheath parameters are independent of η_w provided it is more negative than the sheath potential ($\eta_w > \eta_s$) and $\eta_w = 3$ is big enough for this.

Figure 1 shows the calculated ion distribution functions along the presheath for different equivalent viscosity contributions ($\alpha = 1.0, 0.5, 0.1$, and 0.0) with zero drift velocity, $U_d = 0$, equal ion and electron temperatures, $T_{i\infty} = T_e$, and pure transport sources, $\sigma_i = 0$. The case $\alpha = 0$ shown in Fig. 1(d) is almost equivalent to that of Bissel and Johnson,[7] who treat ions born within the presheath due to ionization of Maxwellian neutrals whose temperature is the same as that of ions. The difference lies in the fact that they treat a mirror symmetric presheath whereas we treat a semi-infinite one and thus take the distribution coming in from large distances to be Maxwellian. Near the probe our calculated distributions are very similar to theirs. The curves labeled 2 are those characteristic of the mesh point adjacent to the probe. We take this as corresponding to the sheath edge. For sheath of zero thickness (corresponding to $\lambda = 0$) there should be no particles with positive velocity at the sheath edge, because the source in the sheath is zero. However, our solutions show a very small components with $u > 0$. This is because the finite mesh spacing prevents us from giving the distribution any closer to the probe than this mesh point.

Inspection of these results shows that the viscous momentum exchange acts so as to smooth out the distributions. The sharply peaked shapes that arise in the zero viscosity case are removed and the distribution broadened, especially close to the probe. This is, of course, because of our momentum exchange term.

A major interest in this work is the effect of parallel ion drift on the ion collection current to the probe. In order to provide a comparison with fluid theories, and because it helps to compress the results onto universal curves, we define an ion-acoustic speed (C_s) and Mach number (M) for the external drift velocity as follows:

$$C_s \equiv \sqrt{(ZT_e + T_{i\infty})/m_i}, \quad M \equiv -V_d/C_s. \tag{18}$$

Thus C_s is an isothermal acoustic speed based on the ion temperature outside the presheath, and the related drift Mach number M has a sign such that positive values denote flow toward the probe. We also express the current density J_s in units normalized to $n_\infty C_s$.

In Fig. 2(a), the sheath-edge current density (and hence the current density to the probe) is plotted for three different α values as a function of drift velocity. In addition, two corresponding curves from fluid calculations[10,11] are shown. Agreement is reasonable, although the kinetic calculation gives slightly higher currents when the drift is toward the probe. It should be recalled that the fluid models assume constant ion temperature, and impose $M = 1$ at the sheath edge. Our present calculation, by comparison, calculates the full ion distribution function and simply treats the probe sheath as a sink, assuming nothing about the flow velocity at the sheath edge. The larger value of J_s from the kinetic mod-

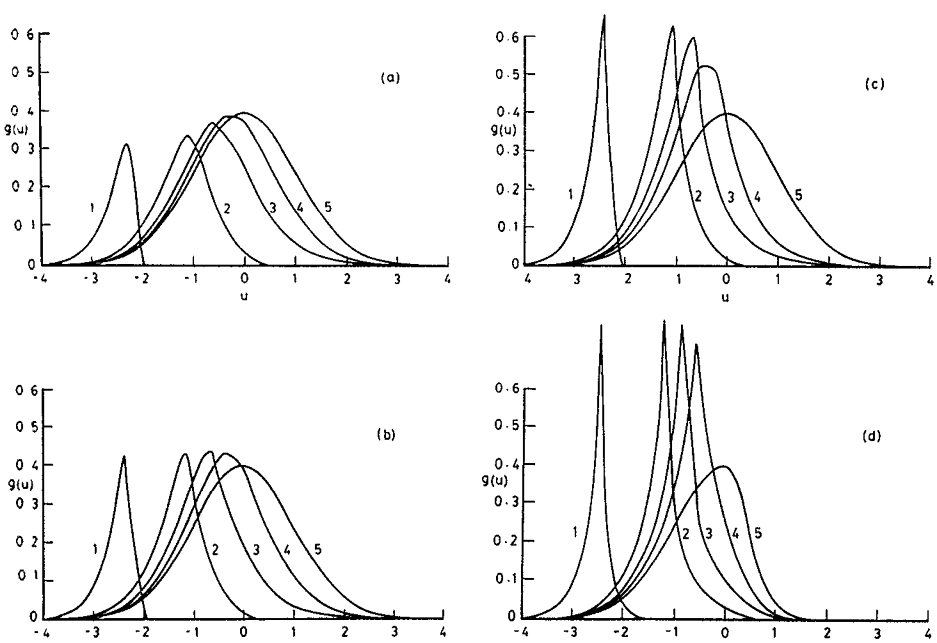

FIG. 1. Ion distributions along the presheath for different viscosity contributions. $\alpha =$ (a) 1.0, (b) 0.5, (c) 0.1, and (d) the $\alpha = 0.0$ case which is approximately equivalent to Ref. 6. Different curves are at different positions corresponding to $\eta = \eta_\omega$ (wall potential; curve 1), η_s (sheath potential; curve 2), $\frac{1}{2}\eta_s$ (curve 3), $\frac{1}{3}\eta_s$ (curve 4), and 0 (curve 5) for $\lambda = 0$, $V_d = 0$, $T_{i\infty} = T_e$, and $\eta_\omega = 3$.

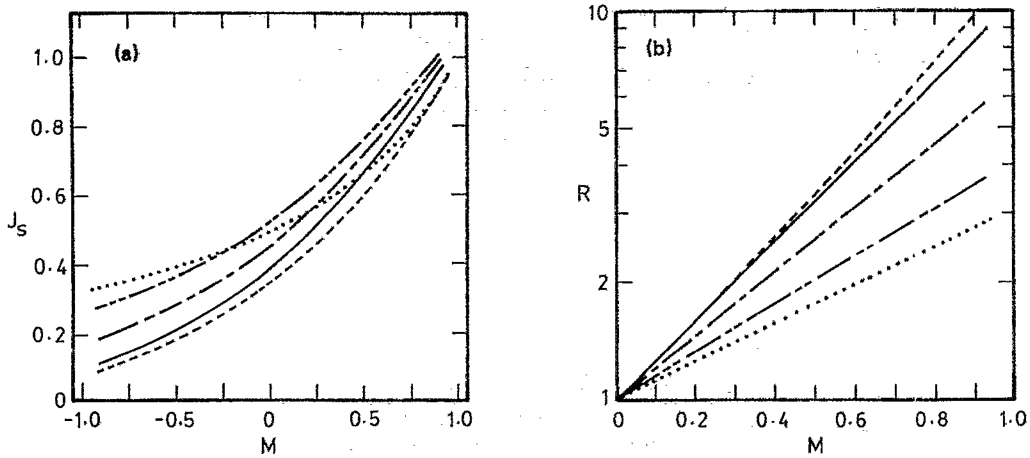

FIG. 2. Sheath current densities (a) and ratios (b) for different viscositycosity contributions. [$\alpha = (- .. -)0.0$], $(- . -)$ 0.5, (—) 1.0, with $-1 < M < 1$ and $T_{i\infty} = T_e$. Comparison is made with fluid models Hutchinson[11] (– – –) and Stangeby[10] (...)].

els may probably be explained as arising from the use of the isothermal acoustic speed corresponding to $T_{i\infty}$ in the velocity normalization. In reality, the ions flow into the sheath at approximately the acoustic speed corresponding to *adiabatic* ions having a *local temperature* different from $T_{i\infty}$. When the drift is toward the probe, the ion temperature is somewhat reduced from $T_{i\infty}$. However, the local acoustic speed is $\sqrt{(ZT_e + \gamma T_i)/m_i}$ with $\gamma = 3$ for one-dimensional ion motion. The combined effect is that the local acoustic speed is rather larger than C_s as defined in Eq. (18) and therefore the current is somewhat larger. By comparison, as will be shown in the last figure, for drifts away from the probe the local ion temperature at the sheath edge is much more strongly reduced, especially for $\alpha = 0$. So much so, it appears, that the local acoustic speed is actually smaller than C_s and so J_s falls below the fluid curve when $M < 0$ and $\alpha = 0$.

In Fig. 2(b) the ratio (R) of upstream to downstream collection current is shown. This is the parameter that is used to deduce drift velocities from Mach probe measurements. The agreement between kinetic and fluid results is very good for $\alpha = 1$, but for $\alpha = 0$ the fluid curve lies somewhat below the kinetic curve because of the J_s behavior just discussed.

We have found that in our solutions, as illustrated in Fig. 2(b), the current ratio is rather accurately proportional to an exponential of the drift velocity. We express this dependence as

$$R = \exp(M/M_c), \tag{19}$$

where M_c is the calibration factor. [It can be related to our previous[13] constant K in an expression $R = \exp(KU_d)$ via $1/M_c = K\sqrt{(1 + T_{i\infty}/ZT_e)}$.]

We show the effect of varying between ionization and transport sources to the presheath by changing the contribution of ionization (σ_i) keeping $\sigma_t + \sigma_i = 1$. Figure 3 gives the collection current and the current ratio for $\sigma_i = 0.1$, 0.3, and 0.5. Here we use the Bissel and Johnson type of source for the ionization by putting $\gamma_{ij} = n_i$, independent of velocity. (Note that we could also recover the Emmert *et al.* case by assuming $\gamma_{ij} = |u_{ij}|$.[16]) The results show that as σ_i increases, J_s increases and R decreases for the same drift velocity. In other words, the larger the contribution of ionization (σ_i), the bigger the ion sheath current density (J_s) and the smaller the current density ratio (R).

The variation of the calibration factor M_c is shown in Fig. 4 for different viscosity contributions $(0 \leqslant \alpha \leqslant 1)$, ioniza-

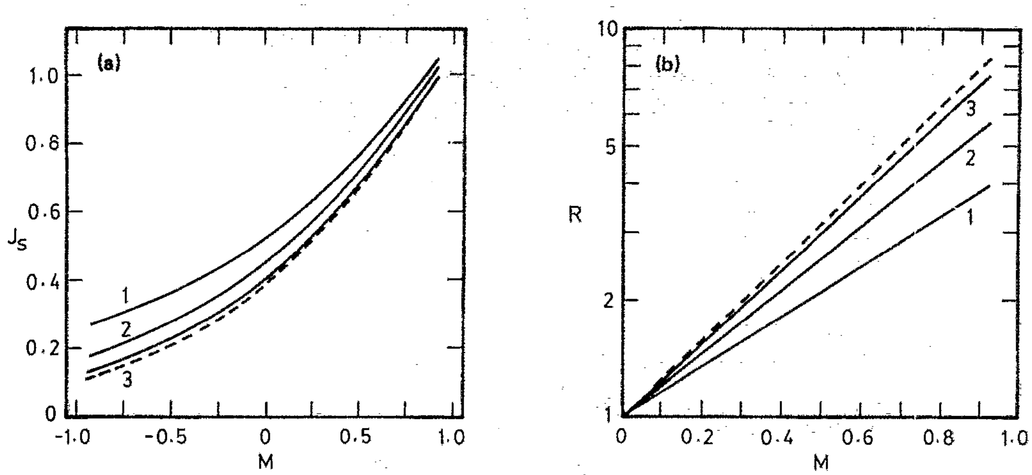

FIG. 3. Effect of ionization. Sheath current densities (a) and ratios (b) are shown for $\sigma_i = 0.1$ (line 3), 0.3 (line 2), and 0.5 (line 1) with $T_{i\infty} = T_e$. The broken line is from the case of $\sigma_i = 0.0$ (no ionization); $\sigma_t + \sigma_i = 1.0$ is applied to all the lines.

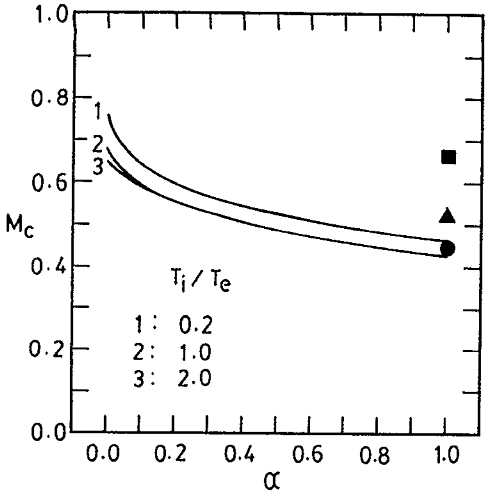

FIG. 4. Variations of the calibration factor (M_c) in the form of $R = \exp(M/M_c)$ for different viscosity contributions ($0 \leqslant \alpha \leqslant 1$), ionization ratios ($0 \leqslant \sigma_i \leqslant 0.5$), and ion temperatures. Curves are for $\sigma_i = 0$ and $T_{i\infty}/T_e = 0.2$ (curve 1), 1.0 (curve 2), 2.0 (curve 3). Points are for $\alpha = 1$ and $T_{i\infty} = T_e$; ●: the case of $\sigma_i = 0.1$, ▲: $\sigma_i = 0.3$, and ■: $\sigma_i = 0.5$; $\sigma_i + \sigma_t = 1.0$ is applied to all the values.

tion ratios ($0 \leqslant \sigma_i \leqslant 0.5$), and ion temperatures ($0.2 \leqslant T_{i\infty}/T_e \leqslant 2.0$). The value of M_c is decreased by stronger viscosity along the presheath, smaller ionization, or larger ion temperature for $T_{i\infty}/T_e \leqslant 1$. However, the ion temperature effect is very weak and for $\alpha > 0.1$, the two lines for $T_{i\infty}/T_e = 1$ and 2 overlap. Thus for the purposes of determining the Mach number, ion temperature variation can be neglected, even though ion temperature affects the *absolute* velocity through its contribution to the sound speed.

Figure 5 shows the variation with the drift velocity (M) and viscosity (α) of ion temperature at the sheath edge, ob-

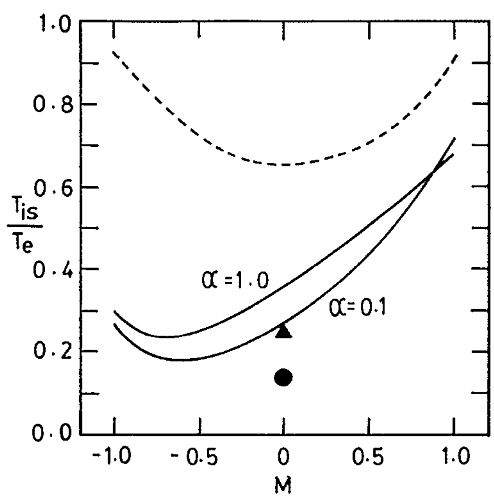

FIG. 5. Sheath ion temperatures (T_{is}) with different drifts (M) and viscosity contributions (α). Solid lines are from the kinetic model for $\alpha = 1.0$ and 0.1, and the broken line is from the fluid model with energy equation for $\alpha = 1.0$,[15] ▲: from Emmert *et al.*,[6] ●: from Bissel and Johnson,[7] which are equivalent to $\sigma_t = 0$ with different ionization source types.

tained from our kinetic model. We show also, for comparison, results from the fluid model of Laux *et al.*[17] whose calculations correspond to $\alpha = 1$. They used an energy equation approximated from the three-dimensional energy equation. Hence the ratio of specific heats (γ) is 5/3 in their case, while it is effectively 3 in ours. This probably explains the difference in ion temperature variations, and hence in ion temperatures at the sheath, especially for the downstream drifts. We also show the sheath temperatures from other kinetic models without drift.

V. CONCLUSIONS

We have obtained self-consistent solutions of the one-dimensional Boltzmann/Poisson equations for the presheath, and given moments such as current density and temperature. We have shown the effects of viscosity and ionization in a strongly magnetized plasma by using a generalized source term. As the contribution of viscosity becomes larger, ion distributions near the probe surface become broader, and sheath current densities become smaller.

Ion sheath current densities and ratios (R) of upstream to downstream current have been presented as a function of normalized plasma drift velocity (M), equivalent viscosity ratio (α), ion temperature ($T_{i\infty}$), and ionization rate (σ_i). The Mach probe calibration factor (M_c) has been given for use in the expression $R = \exp(M/M_c)$. It is found that M_c decreases as α increases, $T_{i\infty}$ decreases, or σ_i increases. However, when the cross-field transport sources dominate ($\sigma_i \ll 1$), and shear viscosity is appreciable ($\alpha > 0.2$) the calibration factor is between about 0.45 and 0.6. Comparisons with fluid and kinetic models indicate reasonable agreement, the discrepancies appear to be qualitatively explicable in terms of the ion temperature variation that is omitted from the fluid models, and the different ion contribution to the acoustic speed.

Our kinetic model produces smaller sheath ion temperatures than those of the fluid model of Laux *et al.* which allows ion temperature variation. This seems to be attributable to their use of an energy equation based on three degrees of freedom whereas the present is a purely one-dimensional calculation of the velocity distribution.

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