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Electrostatic instability of electron beams in a planar $E \times B$ amplifier[a]

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A wave equation is kinetically derived for an electron beam to describe the linear electrostatic perturbations propagating in the perpendicular direction with respect to the applied magnetic field in a planar $E \times B$ amplifier in which the operating electric field is inhomogeneous. For $\omega_c > \omega_p$, the massless guiding center limit is taken to obtain the wave equation for the analysis of the electrostatic instability encountered in the planar $E \times B$ amplifier. In this work the plasma density profile is assumed to be a step function with vacuum boundaries between the cathode and the anode. The growth rates of the perpendicular electrostatic wave are obtained for the parameters such as the location and the thickness of the electron beam as well as the wave number. © *2008 American Institute of Physics.* [DOI: 10.1063/1.2805387]

I. INTRODUCTION

Crossed-field devices such as $E \times B$ amplifiers and magnetrons are important high power microwave generators.[1] They consist of a vacuum region inside of which an electron plasma is created. The stability of electron plasmas in such crossed-field configuration has been the subject of wide investigation in connection with the operation of the devices.[2–6] Although the tremendous amount of work has been done on these devices, it is still very difficult to predict with certainty from the theory how well a new device will work because of the complexity of the system and the poorly understood physical processes.[7,8] The purpose of this work is to present a detailed mathematical analysis of the Vlasov-Poisson equations for a planar $E \times B$ amplifier and to show the growth rate of the nonrelativistic, longitudinal (electrostatic) wave propagating perpendicular with respect to the operating magnetic field. From the results of the present work, one can determine how the thickness and location of an electron plasma beam are related to the excitation of electrostatic perpendicular wave.

II. DISPERSION RELATION AND RESULTS

For a kinetic description of electrostatic waves in electron plasmas produced in a device such as a planar magnetron, we use Vlasov and Poisson equations given by

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}_0\right) \cdot \nabla_\mathbf{v} f = 0, \tag{1}$$

$$\nabla \cdot \mathbf{E} = -4\pi e \int_{-\infty}^{-\infty} f d^3 v, \tag{2}$$

where f is the electron distribution function and other notations follow the standard usage in plasma physics. We take the magnetic field to be external and constant in time and space, and given by $\mathbf{B}_0 = -B_0\hat{\mathbf{z}}$. The unperturbed electric field, produced by the anode and cathode of the device, is a function of y and denoted by $\mathbf{E}_0 = -E_0(y)\hat{\mathbf{y}}$. Then $E \times B$ drift by electron plasmas occurs in the $\hat{x}$ direction. Assuming the perturbations to be purely electrostatic, the perturbed (first-order) plasma distribution function can be written as

$$f_1(\mathbf{x},\mathbf{v},t) = -\frac{e}{m}\int_{-\infty}^{t} \nabla'\Phi(\mathbf{x}',\mathbf{v}') \cdot \nabla_{\mathbf{v}'} f_0(\mathbf{x}',\mathbf{v}')dt', \tag{3}$$

where the subscripts 0 and 1 denote the equilibrium and perturbed quantities, respectively. The electric potential Φ, associated with the electric field, is given by

$$\Phi(\mathbf{x},t) = \tilde{\Phi}(y)\exp[ik_\perp x + ik_\parallel z - i\omega t], \tag{4}$$

which is appropriate for a wave propagating in a medium with inhomogeneity along the $\hat{y}$ direction. In Eq. (4), $k_\perp$ and $k_\parallel$ are the $\hat{x}$ and $\hat{z}$ components of the wave number, respectively. Then, for low density plasma ($\omega_c > \omega_{p0}$), the first-order Poisson equation yields

$$\left(-k_\perp^2 - k_\parallel^2 + 4\pi e^2 \int_{-\infty}^{-\infty} \frac{\partial f_0}{\partial E} d^3 v + \frac{d^2}{dy^2}\right)\tilde{\Phi}(y)$$

$$= -4\pi i e^2 \int_{-\infty}^{-\infty} \left(k_\perp \frac{\partial}{\partial p_x} + k_\parallel \frac{\partial}{\partial p_z} + \omega\frac{\partial}{\partial E}\right)f_0 d^3 v$$

$$\times \int_{-\infty}^{0} \tilde{\Phi}(y')\exp[ik_\perp\xi_x + ik_\parallel\xi_z - i\omega\tau]d\tau, \tag{5}$$

where $\tau = t' - t$, and $p_x = m(v_z - \omega_c y)$, $p_z = mv_z$, and $E = \frac{1}{2}m(v_x^2$

$+v_y^2+v_z^2)-e\Phi_0(y)$ are the constants of motion with $\omega_c=eB_0/mc$ and Φ_0 being the cyclotron frequency and the equilibrium electric potential, respectively. Here, ξ_x, ξ_y, and ξ_z are the particle orbits,

$$\xi_x = v_0\tau + \frac{1}{\Delta}(v_x - v_0)\sin\Delta\tau - \frac{\omega_c}{\Delta^2}v_y(\cos\Delta\tau - 1), \quad (6a)$$

$$\xi_y = \frac{1}{\omega_c}(v_x - v_0)(\cos\Delta\tau - 1) + \frac{v_y}{\Delta}\sin\Delta\tau, \quad (6b)$$

$$\xi_z = v_z\tau, \quad (6c)$$

where $v_0 = cE_0(y_0)/B_0$ and $\Delta = (\omega_c^2 - \omega_{p0}^2)^{1/2}$ are the $E \times B$ drift velocity and the reduced cyclotron frequency, respectively, with $\omega_{p0}^2 = 4\pi n_0(y_0)e^2/m$ being the plasma frequency and y_0 the guiding center.

To solve the integrodifferential equation for $\widetilde{\Phi}$, Eq. (5), we shall need an adequate model for the plasma distribution function such as a Maxwellian. We choose to expand $\widetilde{\Phi}(y')$ about the position y and shall ignore the third order or higher terms,

$$\widetilde{\Phi}(y') = \widetilde{\Phi}(y) + \xi_y\frac{d\widetilde{\Phi}}{dy} + \frac{1}{2}\xi_y^2\frac{d^2\widetilde{\Phi}}{dy^2} + O(\xi_y^3). \quad (7)$$

We assume that the Larmor radius is much smaller than the wavelength as well as the scale length of the inhomogeneity, $L = |\nabla E_0/E_0|^{-1}$. We also assume that the thermal velocity is much smaller than the phase velocity of the wave. Then, in the case of perpendicular propagation of the wave ($k_\parallel = 0$), by carrying out the velocity and time integrations in Eq. (5), one obtains, after a long algebra,

$$\frac{d^2\widetilde{\Phi}}{dy^2} - \widetilde{\Phi}\left[k_\perp^2 - \frac{k_\perp}{\omega_c(\omega - k_\perp v_0)}\frac{d\omega_p^2}{dy}\right] = 0. \quad (8)$$

The wave equation, Eq. (8), and the appropriate boundary conditions constitute an eigenvalue equation with the wave frequency being the eigenvalues. If the two planes at $y = 0$ (cathode) and $y = l$ (anode) are perfect conductors, $\widetilde{\Phi}$ must vanish at the planes. Now, let the electron plasma be confined in the region of $0 \le a \le y \le b \le l$, where a and b are the boundary of the plasma and let the number density of plasma be a step function: $n(y) = \bar{n}(a \le y \le b)$ with constant $\bar{n}$. Then the derivative of ω_p^2 with respect to the dimensionless position $\widetilde{y}(=ky)$ can be determined as $d\omega_p^2/d\widetilde{y} = \bar{\omega}_p^2[\delta(\widetilde{y} - \widetilde{a}) - \delta(\widetilde{y} - \widetilde{b})]$, where $\bar{\omega}_p^2(=4\pi\bar{n}e^2/m)$ is a constant, $\widetilde{a}(=ka)$ and $\widetilde{b}(=kb)$ are the dimensionless boundaries of the plasma, and δ is the Dirac delta function. Then the drift velocity can be found as $v_0 = \bar{\omega}_p^2(\widetilde{y} - \widetilde{a})/\omega_c k$, and Eq. (8) reads

$$\frac{d^2\widetilde{\Phi}}{d\widetilde{y}^2} - \widetilde{\Phi}\left\{1 - \frac{1}{\Gamma - \widetilde{y} + \widetilde{a}}[\delta(\widetilde{y} - \widetilde{a}) - \delta(\widetilde{y} - \widetilde{b})]\right\} = 0, \quad (9)$$

where $\Gamma \equiv \omega\omega_c/\bar{\omega}_p^2$. Satisfying the boundary conditions, $\widetilde{\Phi}(0) = \widetilde{\Phi}(l) = 0$, the solution of Eq. (11) can be found for the following three regions:
    region I ($0 \le \widetilde{y} \le \widetilde{a}$):

$$\widetilde{\Phi}_I(\widetilde{y}) = \left(\frac{d\widetilde{\Phi}}{d\widetilde{y}}\right)_{\widetilde{y}=0}\sinh\widetilde{y}, \quad (10)$$

region II ($\widetilde{a} \le \widetilde{y} \le \widetilde{b}$):

$$\widetilde{\Phi}_{II}(\widetilde{y}) = Ce^{\widetilde{y}} + De^{-\widetilde{y}}, \quad (11)$$

region III ($\widetilde{b} \le \widetilde{y} \le \widetilde{l}$):

$$\widetilde{\Phi}_{III}(\widetilde{y}) = Fe^{\widetilde{y}} + Ge^{-\widetilde{y}}. \quad (12)$$

Here, the constants C, D, F, and G are given by

$$C = \frac{1}{2}e^{-\widetilde{a}}\left(\frac{d\widetilde{\Phi}}{d\widetilde{y}}\right)_{\widetilde{y}=0}\left[\left(1 - \frac{1}{\Gamma}\right)\sinh\widetilde{a} + \cosh\widetilde{a}\right], \quad (13a)$$

$$D = \frac{1}{2}e^{\widetilde{a}}\left(\frac{d\widetilde{\Phi}}{d\widetilde{y}}\right)_{\widetilde{y}=0}\left[\left(1 + \frac{1}{\Gamma}\right)\sinh\widetilde{a} - \cosh\widetilde{a}\right], \quad (13b)$$

$$F = \frac{1}{4}e^{-\widetilde{a}}\left(\frac{d\widetilde{\Phi}}{d\widetilde{y}}\right)_{\widetilde{y}=0}\left\{2\left(1 - \frac{e^{-\widetilde{a}}\sinh\widetilde{a}}{\Gamma}\right) + \frac{1}{\Gamma - \widetilde{b} + \widetilde{a}}\right.$$
$$\left.\times\left[1 - \frac{e^{-\widetilde{a}}\sinh\widetilde{a}}{\Gamma} - e^{-2\widetilde{b}}\left(1 - \frac{e^{\widetilde{a}}\sinh\widetilde{a}}{\Gamma}\right)\right]\right\} \quad (13c)$$

$$G = -\frac{1}{4}e^{-\widetilde{a}}\left(\frac{d\widetilde{\Phi}}{d\widetilde{y}}\right)_{\widetilde{y}=0}\left\{2\left(1 - \frac{e^{\widetilde{a}}\sinh\widetilde{a}}{\Gamma}\right) + \frac{1}{\Gamma - \widetilde{b} + \widetilde{a}}\right.$$
$$\left.\times\left[-1 + \frac{e^{\widetilde{a}}\sinh\widetilde{a}}{\Gamma} + e^{2\widetilde{b}}\left(1 - \frac{e^{-\widetilde{a}}\sinh\widetilde{a}}{\Gamma}\right)\right]\right\}. \quad (13d)$$

Applying the boundary conditions to Eq. (12) gives a quadratic equation for Γ such as $P\Gamma^2 - 2Q\Gamma + R = 0$, where $P = 4\sinh\widetilde{l}$, $Q = (b-a)\sinh\widetilde{l} + \sinh(\widetilde{l} - \widetilde{a} - \widetilde{b})\sinh(\widetilde{a} - \widetilde{b})$, and $R = [(2\widetilde{b} - 2\widetilde{a} + 1)\sinh(\widetilde{l} - \widetilde{a}) + \cosh(\widetilde{l} + \widetilde{a} - 2\widetilde{b})]\sinh\widetilde{a}$. If the discriminant of the quadratic equation is negative, the unstable wave can be developed. Under this condition, the growth rate is

$$\Gamma_i = \gamma\omega_c/\bar{\omega}_p^2 = \sqrt{PR - Q^2}/P, \quad (14)$$

where γ is the imaginary parts of the wave frequency ω. Such instability can occur when the electrons stay in the same phase of the wave, i.e., when the Doppler shifted frequency, $\omega - kv_0$, is zero or nearly zero.

Figure 1 shows the change of the growth rates Γ_i with the scaled distance $\widetilde{l}(=k_\perp l)$ between the anode and the cathode for three different beam thicknesses. We observe that the excited wave appears rapidly at some value of $\widetilde{l}$ and the frequency becomes constant as the wave number increases. Figure 2 depicts the growth rate with the scaled length for three different positions of the single-sized beam ($\widetilde{b} - \widetilde{a} = 0.2$). The general tendency is seen that the growth rate decreases as the beam moves toward the cathode. The variation of the growth rate with the beam thickness $\widetilde{t}(=\widetilde{b} - \widetilde{a})$ is also illustrated in Fig. 3. In Fig. 3, the beam is located on the center between the two electrodes. We see that for a given $\widetilde{l}$ the excited wave disappears as the beam is getting thicker.
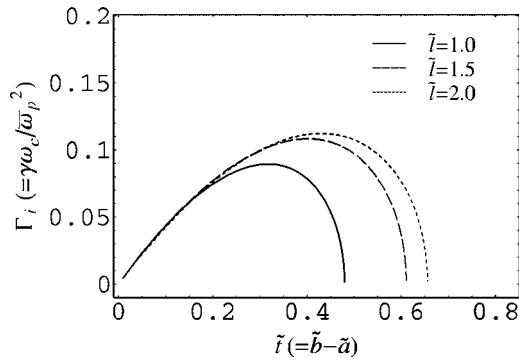
FIG. 1. The growth rate with the scaled length for different electron beam thicknesses.

III. SUMMARY

In this work, the perpendicular wave equation for the drifting electron beam in a planar $E \times B$ amplifier with the inhomogeneous operating electric field is kinetically investigated. The plasma density profile of the electron plasma beam is described by a step function. The cold plasma and massless guiding center limits are assumed in the analysis.
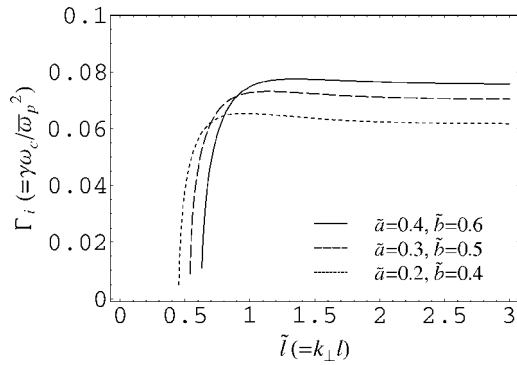

FIG. 2. The growth rate with the scaled length for different electron beam locations. The beam thickness is fixed to $\tilde{b} - \tilde{a} = 0.2$.
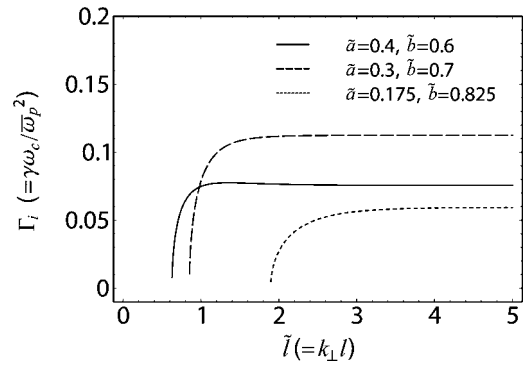

FIG. 3. The growth rate with the electron beam thickness $\tilde{t} (= \tilde{b} - \tilde{a})$ for different scaled lengths. The beam is centered between the cathode and the anode.

We find that the perpendicular electrostatic wave can be excited by the resonant electrons at the phase velocity of the wave. The growth rate of the wave can be maximized by variations of the beam thickness and location. The result of this work would be applicable to optimization of the crossed-field amplifiers provided the relevant physical parameters such as the number density, the strength of the magnetic field, etc., are given.

[1] J. T. Coleman, *Microwave Devices* (Reston, Virgina, 1982).
[2] O. Buneman, R. H. Levy, and L. M. Linson, J. Appl. Phys. **37**, 3203 (1966).
[3] R. C. Davidson and K. T. Tang, Phys. Fluids **28**, 1169 (1985).
[4] S. Riyopoulos, Phys. Plasmas **3**, 1137 (1996).
[5] H. J. Lee, D. J. Kaup, and G. E. Thomas, J. Plasma Phys. **4**, 535 (1988).
[6] D. J. Kaup, Phys. Plasmas **11**, 3151 (2004).
[7] D. J. Kaup, Phys. Plasmas **3**, 771 (1996).
[8] D. J. Kaup, Phys. Plasmas **13**, 053113 (2006).